Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 18 May 2023

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P 7 2 8 6 9 A 0 1 3 2

Pearson

1. The continuous curve C has equation $y = f(x)$.

A table of values of x and y for $y = f(x)$ is shown below.

Interval btwn two x-coords = h of trapezium

x	4.0	4.2	4.4	4.6	4.8	5.0
y	9.2	8.4556	3.8512	5.0342	7.8297	8.6

y_1 y_6

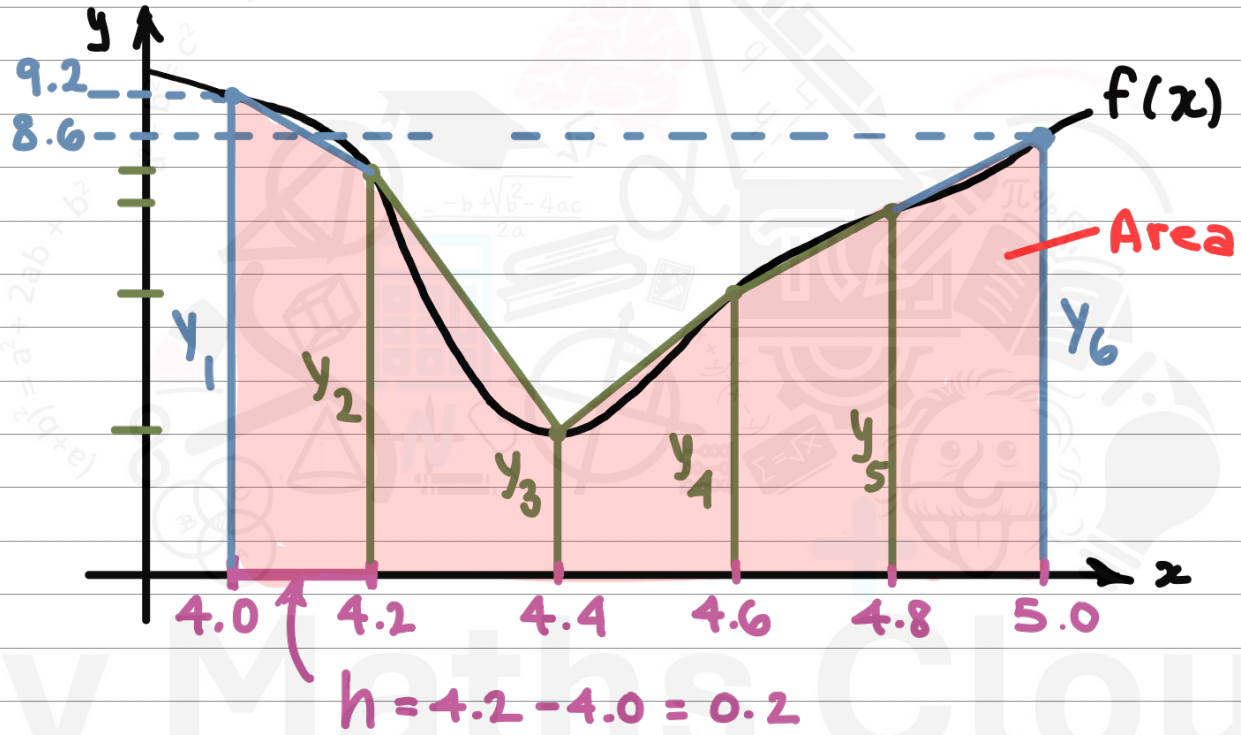
y_1 multiplied by 2 as all are shared by two trapezia

Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_4^5 f(x)\,dx$$

giving your answer to 3 decimal places.

(3)



(Formula is given in formula booklet. The graph is only a visual representation. DO NOT DRAW A GRAPH IN THE EXAM)

$$A = \frac{1}{2}h\left[y_1 + 2(y_2 + y_3 + y_4 + y_5) + y_6\right]$$

Question 1 continued

$$A = \frac{1}{2}(0.2)\left[9.2 + 2(8.4456 + 3.8512 + 5.0342 + 7.8297) + 8.6\right] = 6.81414$$

So : $\int_4^5 f(x)\,dx \approx 6.81414$

$= 6.814$ (3 decimal places)

(Total for Question 1 is 3 marks)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 - 8x^2 + 5x + a$$

where a is a constant.

Given that $(2x - 3)$ is a factor of $f(x)$,

(a) use the factor theorem to show that $a = -3$

(2)

(b) Hence show that the equation $f(x) = 0$ has only one real root.

(4)

(a) ① set the factor equal to 0 :

$2x - 3 = 0$

$x = \frac{3}{2}$

② substitute x into the function to find a :

$4x^3 - 8x^2 + 5x + a = 0$

$4\left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 + 5\left(\frac{3}{2}\right) + a = 0$

$\frac{27}{2} - 18 + \frac{15}{2} + a = 0$

$3 + a = 0 \rightarrow \boxed{a = 3}$

(b) Since $f(x) = (2x-3)$(another expression), use algebraic division to find it :

$2x - 3 \overline{\big)\, 4x^3 - 8x^2 + 5x - 3 = 0}$

(1) Divide first term of dividend by first term of divisor : $\frac{4x^3}{2x} = 2x^2$

$2x^2$

$2x - 3 \overline{\big)\, 4x^3 - 8x^2 + 5x - 3}$

$\div$

↳ $2x^2$ is the first term of our quotient.

Question 2 continued

(2) Multiply out and write under:

$$
\begin{array}{r|l}
 & 2x^2 \\
2x-3 & 4x^3-8x^2+5x-3 \\
 & 4x^3-6x^2 \\
\hline
\end{array}
$$

(3) Subtract then bring down next term:

$$
\begin{array}{r|l}
 & 2x^2 \\
2x-3 & 4x^3-8x^2+5x-3 \\
 & -(4x^3-6x^2) \quad \downarrow \\
\hline
 & -2x^2+5x
\end{array}
$$

(4) Divide first term of expression by first term of divisor: $\frac{-2x^2}{2x} = -x$

$$
\begin{array}{r|l}
 & 2x^2-x \\
2x-3 & 4x^3-8x^2+5x-3 \\
 & -(4x^3-6x^2) \\
\hline
 & -2x^2+5x
\end{array}
$$

(5) Multiply out and write under:

$$
\begin{array}{r|l}
 & 2x^2-x \\
2x-3 & 4x^3-8x^2+5x-3 \\
 & -(4x^3-6x^2) \\
\hline
 & -2x^2+5x \\
 & 2x^2+3x
\end{array}
$$

(6) Subtract then bring down next term:

$$
\begin{array}{r|l}
 & 2x^2-x \\
2x-3 & 4x^3-8x^2+5x-3 \\
 & -(4x^3-6x^2) \\
\hline
 & -2x^2+5x \\
 & -(2x^2+3x) \\
\hline
 & 2x-3
\end{array}
$$

(Total for Question 2 is 6 marks)

(7) Divide first term of expression by first term of divisor : $\frac{2x}{2x} = 1$

$$
\begin{array}{r|l}
 & 2x^2 - x + 1 \\
\hline
2x-3 & 4x^3 - 8x^2 + 5x - 3 \\
 & -(4x^3 - 6x^2) \\
\hline
 & -2x^2 + 5x \\
 & -(2x^2 + 3x) \\
\hline
 & 2x - 3
\end{array}
$$

(8) Multiply out and write under :

$$
\begin{array}{r|l}
 & 2x^2 - x + 1 \\
\hline
2x-3 & 4x^3 - 8x^2 + 5x - 3 \\
 & -(4x^3 - 6x^2) \\
\hline
 & -2x^2 + 5x \\
 & -(2x^2 + 3x) \\
\hline
 & 2x - 3 \\
 & 2x - 3
\end{array}
$$

(9) Subtract :

$$
\begin{array}{r|l}
 & 2x^2 - x + 1 \\
\hline
2x-3 & 4x^3 - 8x^2 + 5x - 3 \\
 & -(4x^3 - 6x^2) \\
\hline
 & -2x^2 + 5x \\
 & -(2x^2 + 3x) \\
\hline
 & 2x - 3 \\
 & -(2x - 3) \\
\hline
 & 0
\end{array}
$$

$f(x) = (2x-3)(2x^2 - x + 1)$

discriminant $= (b^2 - 4ac)$

$= (-1)^2 - 4(2)(1) = -7 < 0$ so no real roots for $(2x^2 - x + 1)$

so : $f(x) = 0$ has only one root, $x = \frac{-3}{2}$

3. A circle C has centre (2, 5)

Given that the point $P(8, -3)$ lies on C

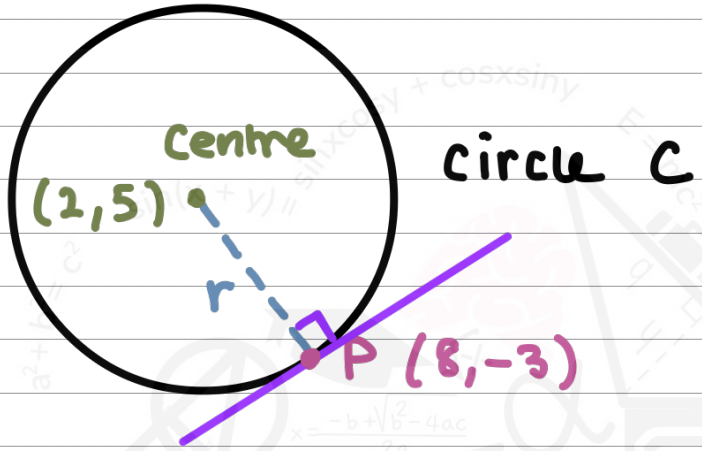
(a) (i) find the radius of C

(ii) find an equation for C **(3)**

$m_{tangent} = -\dfrac{1}{m_{radius}}$

(b) Find the equation of the tangent to C at P giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found. **(4)**

(a)(i)



Eq. of circle :

$$(x - x_{centre})^2 + (y - y_{centre})^2 = r^2$$

$$(8 - 2)^2 + ((-3) - 5)^2 = r^2$$

$$36 + 64 = r^2$$

$$\sqrt{100} = \sqrt{r^2}$$

$$r = 10$$

(ii) write found r into the equation :

$$(x - 2)^2 + (y - 5)^2 = 10^2$$

you can keep it as-is or write it as 100.

for any point on circle C.

Question 3 continued

(b) $y = mx + c$

$m = \frac{2 - 8}{5 - (-3)} = \frac{-6}{8} = \frac{-4}{3} \rightarrow m_{radius}$

The tangent is perpendicular to the radius, so:

$m_{radius} = \frac{-4}{3}$ then $m_{tangent} = \frac{3}{4}$

$y = \frac{3}{4}x + c$

Plug in $(8, -3)$ to find c

$-3 = \frac{3}{4}(8) + c \rightarrow c = -9$

$\rightarrow y = \frac{3}{4}x - 9$

The q asked for the answer in the form:

$\rightarrow ax + by + c$ so:

$4 \times (y) = \left(\frac{3}{4}x - 9\right) \times 4$

$\cancel{4y} = 3x - 3$

$-\cancel{4y} \quad -4y$

$3x - 4y - 3 = 0$

(Total for Question 3 is 7 marks)

4. The binomial expansion, in ascending powers of x, of

$$(3+px)^5 = (a+bx)^n$$

where p is a constant, can be written in the form

$$A + Bx + Cx^2 + Dx^3 \ldots$$

where A, B, C and D are constants.

(a) Find the value of A

(1)

Given that

- $B = 18D$
- $p < 0$

(b) find

(i) the value of p

(ii) the value of C

(6)

BONUS NOTE: BINOMIAL COEFFICIENTS ARE SYMMETRIC

$(3+px)^5$ has six terms, where:

$${}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$$

where: ${}^5C_0 = {}^5C_5$, ${}^5C_1 = {}^5C_4$, ${}^5C_2 = {}^5C_3$

(a) Binomial expansion formula:

$$\boxed{(a+bx)^n = {}^nC_0 a^n (bx)^0 + {}^nC_1 a^{n-1}(bx)^1 + {}^nC_2 a^{n-2}(bx)^2 + \ldots}$$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

n: power of binomial

r: (position of term) − 1

A is the first term:

$$A = {}^5C_0\, 3^5 (px)^0$$

$$A = \frac{5!}{(5-0)!\,0!}(3^5)(1)$$

$$A = \underbrace{\frac{120}{(5)!\;1}}(243)(1) = (1)(243)(1) = 243$$

$$A = 243$$

FACTORIALS

$x! = x(x-1)(x-2)\ldots(1)$

$3! = 3\times2\times1 = 6$

$5! = 5\times4\times3\times2\times1 = 120$

$0! = 1$

NOTE: FIRST AND LAST ${}^nC_r$ ARE ALWAYS $= 1$.

Question 4 continued

(b)(i) $A + Bx + Cx^2 + Dx^3$

$247 + Bx + Cx^2 + Dx^3$

① Find B :

Bx is the second term :

$Bx = {}^5C_1\, 3^{5-1} (px)^1$

$Bx = \frac{5!}{(5-1)!\,1!}(3^4)(px)$

$Bx = \frac{120}{(4)!(1)}(81)\,px = \frac{120}{24(1)}(81)\,px$

$Bx = 405\,px$

$B = 405p$

② Find D :

Dx^3 is the fourth term :

$Dx^3 = {}^5C_3\, 3^{5-3} (px)^3$

$Dx^3 = \frac{5!}{(5-3)!\,3!}(3^2)(px)^3$

$Dx^3 = \frac{120}{(2)!(6)}(9)(px)^3 = \frac{120}{2(6)}(9)p^3x^3$

$Dx^3 = 90p^3x^3$

$D = 90p^3$

Since $B = 405p$

$B = 18D$

$D = 90p^3$

Question 4 continued

$405p = 18(90)p^3$

$\frac{1}{p} \times (405p) = 18(90)p^3 \times \frac{1}{p}$

$\frac{405}{1620} = \frac{1620p^2}{1620}$

$\sqrt{p^2} = \sqrt{\frac{1}{4}} \rightarrow p = \pm \frac{1}{2}$

$p = -\frac{1}{2}$ since $p < 0$ (stated in q)

(b)(ii)

Cx^2 is the third term :

$Cx^2 = {}^5C_2\, 3^{5-2} (px)^2$

$Cx^2 \quad \frac{5!}{(5-2)!\,2!}(3^3)(px)^2$

$Cx^2 \quad \frac{120}{(3)!(2)}(27)(px)^2 = \frac{120}{(6)(2)}(27)p^2x^2$

$Cx^2 = 270p^2x^2$

$C = 270p^2$

$C = 270\left(\frac{-1}{2}\right)^2 = \frac{135}{2}$

$C = \frac{135}{2}$

Question 4 continued

(Total for Question 4 is 7 marks)

5. Use the laws of logarithms to solve

$$\log_2(16x) + \log_2(x+1) = 3 + \log_2(x+6)$$

(5)

$$\log_2(16x) + \log_2(x+1) = 3 + \log_2(x+6)$$

• Recall the rules of logarithms :

$$\log_a(b) + \log_a(c) = \log_a(b \times c)$$

$$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$$

① rearrange the equation :

$$\log_2(16x) + \log_2(x+1) = 3 + \cancel{\log_2(x+6)}$$

$$-\log_2(x+6) \qquad \cancel{-\log_2(x+6)}$$

$$\log_2(16x) + \log_2(x+1) - \log_2(x+6) = 3$$

② use the rules of logarithms :

$$\log_2(16x) + \log_2(x+1) - \log_2(x+6) = 3$$

$$\log_2\left(16x \times (x+1)\right) - \log_2(x+6) = 3$$

$$\log_2\left(\frac{16x(x+1)}{x+6}\right) = 3$$

③ Use the rule :

$$\log_2 a = b \longrightarrow 2^b = a$$

$$\log_2\left(\frac{16x(x+1)}{x+6}\right) = 3$$

so : $\dfrac{16x(x+1)}{x+6} = 2^3$

Question 5 continued

④ multiply to get rid of denominator :

$$\cancel{(x+6)}\left(\frac{16x(x+1)}{\cancel{x+6}}\right) = 8(x+6)$$

$$16x(x+1) = 8(x+6)$$

⑤ expand both sides :

$$16x(x+1) = 8(x+6)$$
$$16x^2 + 16x = 8x + 48$$

⑥ Combine like terms and move all terms to one side :

$$16x^2 + 16x = \cancel{8x + 48}$$
$$-8x - 48 \quad \cancel{-8x - 48}$$

$$16x^2 + 16x - 8x - 48 = 0$$
$$16x^2 + 8x - 48 = 0$$

this is a quadratic equation, hence ;

⑦ Factorise quadratic equation :

$$[16x^2 + 8x - 48 = 0] \div 8$$
$$2x^2 + x - 6 = 0$$
$$(2x-3)(x+2) = 0$$

⑧ Solve for x :

$2x - 3 = 0$ → $x = \frac{3}{2}$

$x + 2 = 0$ → $x = -2$ → REJECTED ;

$\log_2(16(-2)) = \log_2(-32)$

$\log_2(-2-1) = \log_2(-3)$

and log of a negative value is not possible.

so : $x = \frac{3}{2}$

(Total for Question 5 is 5 marks)

ALTERNATIVE SOLUTIONS:

① rearrange the equation:

$$\cancel{\log_2(16x)} + \cancel{\log_2(x+1)} = 3 + \log_2(x+6)$$

$$-\log_2(16x) - \log_2(x+1) \qquad -\log_2(x+1) - \log(16x)$$

$$0 = \cancel{3} + \log_2(x+6) - \log_2(16x) - \log_2(x+1)$$

$$-3 - \cancel{-3}$$

$$3 = \log_2(x+6) - \log_2(16x) - \log_2(x+1)$$

② use the rules of logarithms:

$$\left(-3 = \log_2(x+6) - \log_2(16x) - \log_2(x+1)\right)$$

$$\log_2(x+6) - \log_2(16x) - \log_2(x+1) = -3$$

$$\log_2\left(\frac{x+6}{16x}\right) - \log_2(x+1) = -3$$

$$\log_2\left(\frac{x+6}{16x(x+1)}\right) = -3$$

③ Use the rule: $\log_2 a = b \longrightarrow 2^b = a$

$$\log_2\left(\frac{x+6}{16x(x+1)}\right) = -3$$

$$\frac{x+6}{16x(x+1)} = 2^{-3}$$

$$\frac{x+6}{16x(x+1)} = \frac{1}{2^3}$$ (cross multiply)

$$16x(x+1) = 8(x+6)$$

$$16x^2+16x = 8x+48$$

$$-8x-48x \qquad -8x-48$$

$$[16x^2+8x-48x=0] \div 8$$

$$2x^2+x-6x=0$$

$$(2x-3)(x+2)=0$$

$x=-2$ ✗ reject (reason in first solution)

$$x=\frac{3}{2}$$

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A software developer released an app to download.

multiply term value x1000 to find no. of downloads

The numbers of downloads of the app each month, in thousands, for the first three months after the app was released were

$$2k - 15 \qquad k \qquad k + 4$$

where k is a constant.

Given that the numbers of downloads each month are modelled as a geometric series,

has common ratio (r).

(a) show that $k^2 - 7k - 60 = 0$ **(2)**

(b) predict the number of downloads in the 4th month.

sum term 4 (a_4) **(4)**

The **total** number of all downloads of the app is predicted to exceed 3 million for the first time in the Nth month.

Sum up to a_N > 3,000,000 ÷ 1000 > 3000

(c) Calculate the value of N according to the model. **(3)**

(a)

$2k-15 \xleftrightarrow{\text{common ratio (r)}} k \xleftrightarrow{\text{common ratio (r)}} k+4$

$a_1 \qquad a_2 \qquad a_3$

$r = \frac{a_2}{a_1} = \frac{k}{2k-15} \qquad r = \frac{a_3}{a_2} = \frac{k+4}{k}$

$r = r \rightarrow$ (common ratio is equal between any two consecutive terms)

$\frac{a_2}{a_1} = \frac{a_3}{a_2}$

$\frac{k}{2k-15} = \frac{k+4}{k}$ (cross multiply)

(multiply brackets) $k(k) = (2k-15)(k+4)$

(combine like terms) $k^2 = 2k^2 + 8k - 15k - 60$

$\cancel{k^2} = 2k^2 - 7k - 60$

$-\cancel{k^2} \qquad -k^2$

$0 = k^2 - 7k - 60$

Question 6 continued

(b) (i) factorise $k^2 - 7k - 60 = 0$ to find k :

$$k^2 - 7k - 60 = 0$$
$$(k-12)(k+5) = 0$$

$k - 12 = 0$ or $k + 5 = 0$

$k = 12$ or $k = -5 \rightarrow$ [REJECT ; k cannot be -ve. If $k < 0$, all terms become -ve, no. of downloads can't be -ve.]

(2) Find a_4 :

$a_1 = 2(12) - 15 = 9$

$a_2 = 12$

$a_3 = 12 + 4 = 16$

$r = \dfrac{12}{2(12)-15} = \dfrac{4}{3}$

So the series looks like : $9 \xrightarrow[\times 4/3]{+} 12 \xrightarrow[\times 4/3]{+} 16 + \dots$

Use formula : $a_n = a_1 r^{n-1}$ to find a_4

$a_4 = 9 \times \left(\frac{4}{3}\right)^{4-1} = \frac{64}{3} \times 1000$

$= 21\,300$ (3 sf)

each term is in thousands

(c) $a_1 + a_2 + a_3 + a_4 \dots + a_N > 3\,000$

Use partial sum formula :

$$S_N = \frac{a_1(1 - r^N)}{(1 - r)}$$

Question 6 continued

$$S_N = \frac{9\left(1-\left(4/3\right)^N\right)}{\left(1-4/3\right)} > 3000$$

$$\frac{1}{3} \times \frac{9\left(1-\left(4/3\right)^N\right)}{-1/3} > 3000 \times \frac{1}{3}$$

$$-9\left(1-\left(4/3\right)^N\right) > 1000$$

$$-9 + 9\left(\frac{4}{3}\right)^N > 1000$$

$(+9)$ $(+9)$

$$9\left(\frac{4}{3}\right)^N > 1009$$

- take log of both sides!

$$\log 9\left(\frac{4}{3}\right)^N > \log 1009$$

- USE RULE : $\log a^b = b\log a$

$$\log\left(\frac{4}{3}\right)^N > \log\frac{1009}{9}$$

- Divide both sides to make N subject :

$$N\log\left(\frac{4}{3}\right) > \log\left(\frac{1009}{9}\right)$$

$$\frac{N\log\left(\frac{4}{3}\right)}{\log\left(\frac{4}{3}\right)} > \frac{\log\left(\frac{1009}{9}\right)}{\log\left(\frac{4}{3}\right)}$$

!!

NOTE : $\log(4/3)$ is positive, so no need to flip the inequality ☺

Question 6 continued

$$N > \frac{\log\left(\frac{1009}{9}\right)}{\log\left(\frac{4}{3}\right)}$$

input into calculator as-is.

$N > 16.405$

so $N = 17$

(next whole no. after 16.4 is 17)

(Total for Question 6 is 9 marks)

7. The height of a river above a fixed point on the riverbed was monitored over a 7-day period.

The height of the river, H metres, t days after monitoring began, was given by

$$H = \frac{\sqrt{t}}{20}(20 + 6t - t^2) + 17 \qquad 0 \leqslant t \leqslant 7$$

Given that H has a stationary value at $t = \alpha$ $\quad \frac{dH}{dt} = 0$

(a) use calculus to show that α satisfies the equation

$$5\alpha^2 - 18\alpha - 20 = 0$$

(5)

(b) Hence find the value of α, giving your answer to 3 decimal places.

(1)

(c) Use further calculus to prove that H is a maximum at this value of α. $\quad \frac{d^2H}{dt^2} < 0$

(2)

(a) ① Rewrite :

$$H = \frac{t^{1/2}}{20}(20 + 6t - t^2) + 17$$

multiply $\frac{t^{1/2}}{20}$ into the brackets

$$H = \frac{\cancel{20}t^{1/2}}{\cancel{20}} + \frac{t^{1/2}(6t)}{20} - \frac{t^{1/2}(t^2)}{20} + 17$$

$$\boxed{a^b \times a^c = a^{b+c}}$$

$$H = t^{1/2} + \frac{\cancel{6}^{3}t^{3/2}}{\cancel{20}_{10}} - \frac{t^{5/2}}{20} + 17$$

$$H = t^{1/2} + \frac{3t^{3/2}}{10} - \frac{t^{5/2}}{20} + 17$$

DIFFERENTIATION

$$y = ax^n$$

$$\frac{dy}{dx} = nax^{n-1}$$

② DIFFERENTIATE :

$$\frac{dH}{dt} = \frac{1}{2}t^{(1/2-1)} + \frac{3}{2}\left(\frac{3}{10}\right)t^{(3/2-1)} + \frac{5}{2}\left(\frac{1}{20}\right)t^{(5/2-1)}$$

$$\frac{dH}{dt} = \frac{1}{2}t^{-1/2} + \frac{9}{20}t^{1/2} - \frac{1}{8}t^{3/2} = 0 \quad \text{(Stationary)}$$

③ Substitute $t = \alpha$, simplify :

Question 7 continued

$\frac{1}{2}\alpha^{-1/2} + \frac{9}{20}\alpha^{1/2} - \frac{1}{8}\alpha^{3/2} = 0$ → multiply each term by $\alpha^{1/2}$

$\frac{1}{2}\alpha^{-1/2}(\alpha^{1/2}) + \frac{9}{20}\alpha^{1/2}(\alpha^{1/2}) - \frac{1}{8}\alpha^{3/2}(\alpha^{1/2}) = 0(\alpha^{1/2})$

$\frac{1}{2} + \frac{9}{20}\alpha - \frac{1}{8}\alpha^2 = 0$

$\frac{1}{2}(40) + \frac{9}{20}\alpha(40) + \frac{1}{8}\alpha^2(40) = 0(40)$

LCM OF 2, 8, 20 is 40

$-1 \times (20 + 18\alpha - 5\alpha^2) = 0 \times -1$

$5\alpha^2 - 18\alpha - 20 = 0$

(b) $\underbrace{5}_{a}\alpha^2 \underbrace{- 18}_{b}\alpha \underbrace{- 20}_{c} = 0$

$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(5)(-20)}}{2(5)}$

$\alpha = \frac{9 - \sqrt{181}}{5} = -0.891$ ✗ reject; time can't be negative.

$\alpha = \frac{9 + \sqrt{181}}{5} = 4.491$

(c) $\frac{d^2H}{dt^2} < 0$ at maximum point.

$\frac{dH}{dt} = \frac{1}{2}t^{-1/2} + \frac{9}{20}t^{1/2} - \frac{1}{8}t^{3/2}$

$\frac{d^2H}{dt^2} = -\frac{1}{4}t^{-3/2} + \frac{9}{40}t^{-1/2} - \frac{3}{16}t^{1/2}$

$= -\frac{1}{4}(4.491)^{-3/2} + \frac{9}{40}(4.491)^{-1/2} - \frac{3}{16}(4.491)^{1/2} = -0.317$

$-0.317 < 0$ → So α is a max point.

(Total for Question 7 is 8 marks)

8. (i) A student writes the following statement:

"When a and b are consecutive **prime** numbers, $a^2 + b^2$ is never a multiple of 10"

Prove by counter example that this statement is **not** true.

give example that contradicts the statement.

(2)

(ii) Given that x and y are even integers greater than 0 and less than 6, prove by exhaustion, that

whole numbers

try all values to show inequality is true.

$$1 < x^2 - \frac{xy}{4} < 15$$

(3)

(i) primes : 2, 3, 5, 7, 11, 13, ...

square them : $2^2, 3^2, 5^2, 7^2, 11^2, 13^2, \ldots$

$4, 9, 25, 49, 121, 169$

$49 + 121 = 170$

multiple of 10.

So : statement is not true as 7 and 11 are two consecutive prime numbers, and $7^2 + 11^2 = 170$ which is a multiple of 10.

(b)
- $0 < x < 6$
- x is even
- x is a whole no.

- $0 < y < 6$
- y is even
- y is a whole no.

x can be : 2, 4

y can be : 2, 4

so, 4 cases to exhaust :

$$1 < x^2 - \frac{xy}{4} < 15$$

① $x = 4$, $y = 4$

$$4^2 - \frac{(4)(4)}{4} = 12 \rightarrow 1 < 12 < 15 \text{ is true}$$

Question 8 continued

② $x = 4, \quad y = 2$

$4^2 - \frac{(4)(2)}{4} = 14 \rightarrow 1 < 14 < 15$ is true

③ $x = 2, \quad y = 4$

$2^2 - \frac{(2)(4)}{4} = 2 \rightarrow 1 < 2 < 15$ is true

④ $x = 2, \quad y = 2$

$2^2 - \frac{(2)(2)}{4} = 3 \rightarrow 1 < 3 < 15$ is true

So statement is true for all cases.

(Total for Question 8 is 5 marks)

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$3\cos\theta(\tan\theta\sin\theta + 3) = 11 - 5\cos\theta$$

may be written as

quadratic, only $\cos\theta$.

$$3\cos^2\theta - 14\cos\theta + 8 = 0$$

(3)

(b) Hence solve, for $0 < x < 360°$

$$3\cos 2x(\tan 2x\sin 2x + 3) = 11 - 5\cos 2x$$

giving your answers to one decimal place.

(4)

(a) $3\cos\theta(\tan\theta\sin\theta + 3) = 11 - 5\cos\theta$

$3\cos\theta\tan\theta\sin\theta + 9\cos\theta = 11 - 5\cos\theta$ → $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$3\cancel{\cos\theta}\frac{\sin\theta}{\cancel{\cos\theta}}\sin\theta + 9\cos\theta = 11 - 5\cos\theta$

$3\sin^2\theta + 9\cos\theta = 11 - 5\cos\theta$

Use identity: $\sin^2\theta + \cos^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$

$3(1 - \cos^2\theta) + 9\cos\theta = 11 - 5\cos\theta$

$\cancel{3 - 3\cos^2\theta + 9\cos\theta} = 11 - 5\cos\theta$

$-3 + 3\cos^2\theta - 9\cos\theta \quad -3 + 3\cos^2\theta - 9\cos\theta$

$3\cos^2\theta - 14\cos\theta + 8 = 0$

(b) As shown in (a):

$3\cos 2x(\tan 2x\sin 2x + 3) = 11 - 5\cos 2x$

can be written as:

$3\cos^2 2x - 14\cos 2x + 8 = 0$

Question 9 continued

let $x = \cos 2x$:

$3x^2 - 14x + 8 = 0$

$(3x - 2)(x - 4) = 0$

$x = \frac{2}{3}$ or $x = 4$ REJECT; range of x $y = \cos 2x$ is $-1 \leq y \leq 1$

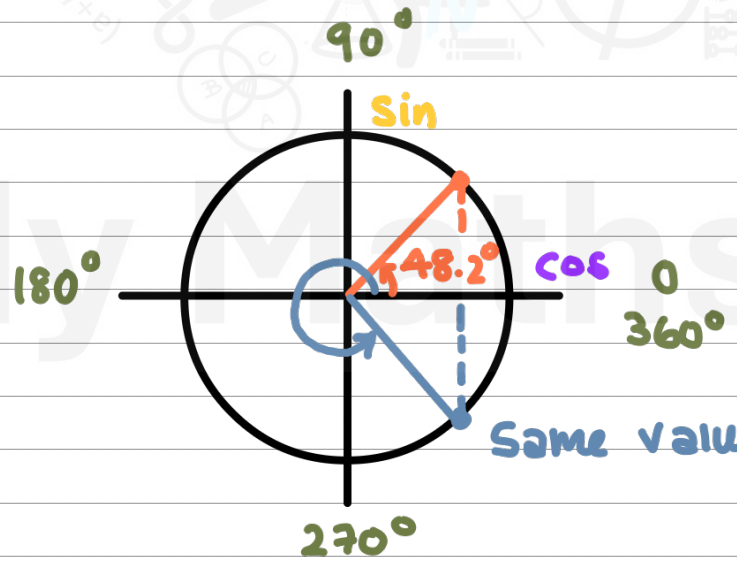
$\cos 2x = \frac{2}{3}$ $\cos 2x = 4$

SO: $\cos 2x = \frac{2}{3}$, $0 < x < 360°$ (×2) $0 < 2x < 720°$

$\cos^{-1}\left(\frac{2}{3}\right) = 48.1896851....$ (on calculator)

$2x = 48.2°$ (1 d.p)

ON UNIT CIRCLE:



Same value for $\cos 2x$.

SO: $2x = 360° - 48.2 = 311.8°$ (1 d.p.)

Question 9 continued

Range : $0 < 2x < 720^\circ$

$2x = 48.2^\circ + 360^\circ = 408.2^\circ$ (1 d.p)

$2x = 311.8^\circ + 360^\circ = 671.8^\circ$ (1 d.p)

$\frac{\cancel{2}x}{\cancel{2}} = \frac{48.2^\circ}{2}, \frac{311.8^\circ}{2}, \frac{408.2^\circ}{2}, \frac{671.8^\circ}{2}$

$x = 24.1^\circ, 155.9^\circ, 204.1^\circ, 335.9^\circ$

Question 9 continued

(Total for Question 9 is 7 marks)

10. The curve C has equation

$$y = \frac{(x-k)^2}{\sqrt{x}} \qquad x > 0$$

cannot be integrated as-is.

where k is a **positive** constant.

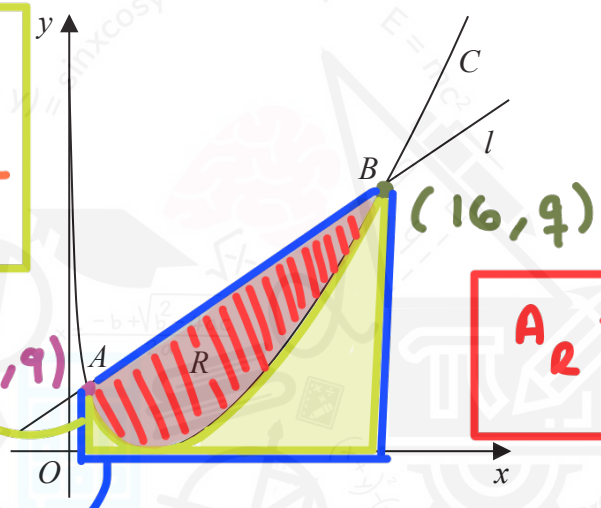
(a) Show that

$$\int_1^{16} \frac{(x-k)^2}{\sqrt{x}}\,dx = ak^2 + bk + \frac{2046}{5}$$

where a and b are integers to be found.

(5)

area given by $\int_1^{16} \frac{(x-k)^2}{\sqrt{x}}\,dx$



$A_R = A_l - A_C$

area under l

Figure 1

Figure 1 shows a sketch of the curve C and the line l.

Given that l intersects C at the point $A(1, 9)$ and at the point $B(16, q)$ where q is a constant,

(b) show that $k = 4$

(2)

The region R, shown shaded in Figure 1, is bounded by C and l

Using the answers to parts (a) and (b),

(c) find the area of region R

(3)

(a) ① expand $(x-k)^2$: $(a+b)^2 = a^2 + 2ab + b^2$

$$y = \frac{x^2 - 2kx + k^2}{\sqrt{x}}$$

② rewrite as 3 separate fractions :

Question 10 continued

$$y = \frac{x^2}{\sqrt{x}} - \frac{2kx}{\sqrt{x}} + \frac{k}{\sqrt{x}}$$

③ rewrite $\sqrt{x}$ as $x^{1/2}$:

$$y = \frac{x^2}{x^{1/2}} - \frac{2kx^1}{x^{1/2}} + \frac{k^2x^0}{x^{1/2}}$$

④ simplify terms : $\frac{a^b}{a^c} = a^{b-c}$

$$y = x^{3/2} - 2kx^{1/2} + k^2x^{-1/2}$$

⑤ Integrate :

$$\int_1^{16} x^{3/2} - 2kx^{1/2} + k^2x^{-1/2}\,dx$$

REMEMBER!

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1}}$$

$$\left[\frac{x^{5/2}}{5/2} - \frac{2kx^{3/2}}{3/2} + \frac{k^2x^{1/2}}{1/2}\right]_1^{16}$$

$$\left[\frac{2}{5}x^{5/2} - \frac{2}{3}(2)kx^{3/2} + 2k^2x^{1/2}\right]_1^{16}$$

$$\left[\frac{2}{5}x^{5/2} - \frac{4}{3}kx^{3/2} + 2k^2x^{1/2}\right]_1^{16}$$

Substitute : $x=16$, $x=1$:

Question 10 continued

$$\left[\frac{2}{5}(16)^{5/2} - \frac{4}{3}k(16)^{3/2} + 2k^2(16)^{1/2}\right] -$$

$$\left[\frac{2}{5}(1)^{5/2} - \frac{4}{3}k(1)^{3/2} + 2k^2(1)^{1/2}\right]$$

$$\left[\frac{2048}{5} - \frac{256}{3}k + 8k^2\right] - \left[\frac{2}{5} - \frac{4}{3}k + 2k^2\right]$$

$$\frac{2048}{5} - \frac{256}{3}k + 8k^2 - \frac{2}{5} + \frac{4}{3}k - 2k^2$$

$$\frac{2048}{5} - \frac{2}{5} + 8k^2 - 2k^2 - \frac{256}{3}k + \frac{4}{3}k$$

$$= \frac{2046}{5} + 6k^2 - \frac{252}{3}k$$

$$= \frac{2046}{5} + 6k^2 - 84k$$

was given in question

(b) Substitute $(1,9)$ into eq. to find k

$$9 = \frac{(1-k)^2}{\sqrt{1}}$$

$$\sqrt{9} = \sqrt{(1-k)^2}$$

$$\pm 3 = 1 - k$$

$k = 1 - 3 = -2$ ✗ reject; k is positive

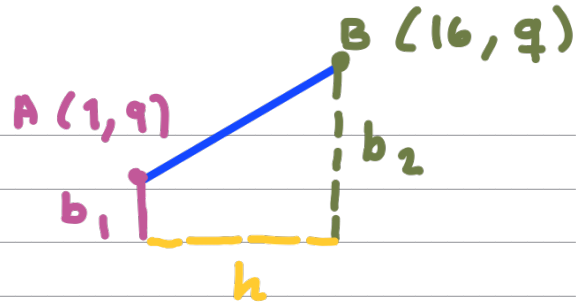
$k = 1 + 3 = 4$

(c) $A_R = A_{\text{under } l} - A_{\text{under } C}$

$$A_l = \frac{1}{2}(b_1 + b_2)h$$

Question 10 continued

$= \frac{1}{2}(9 + q)(16-1)$



FIND q :

$q = \frac{(16-4)^2}{\sqrt{16}} = 36$

$A_\ell = \frac{1}{2}(9 + 36)(15) = \frac{675}{2}$

$A_c = \int_1^{16} \frac{(x-4)^2}{\sqrt{x}}\,dx$

$= \frac{2046}{5} + 6(4)^2 - 84(4) = \frac{846}{5}$

$A_R = \frac{675}{2} - \frac{846}{5} = \frac{1683}{10}$

(Total for Question 10 is 10 marks)

11. A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$\boxed{u_{n+1} = b - au_n}$$

recursive sequence (plugging in a term on the right gives the next term on the left)

$$u_1 = 3$$

starting point

where a and b are constants.

(a) Find, in terms of a and b,

(i) u_2 → $u_2 = u_{1+1}$ $(n=1)$

(ii) u_3 → $u_3 = u_{2+1}$ $(n=2)$

(2)

Given

end

terms : $u_1 + u_2 + u_3 \ldots$

- $\sum_{n=1}^{3} u_n = 153$

sum

starting value

- $b = a + 9$

(b) show that

$$a^2 - 5a - 66 = 0$$

(3)

(c) Hence find the larger possible value of u_2

(3)

(a) This is a recursive sequence :

$u_{n+1} = b - au_n$ → notice how $n+1$ is 1 more than n

Let's find :

(i) u_2 :

$u_2 = b - au_1 = b - a(3) = b - 3a$

(ii) u_3 :

$u_3 = b - au_2 = b - a(b-3a) = b - ab + 3a^2$

Question 11 continued

(b) $\sum_{n=1}^{3} u_n = 153$ means start the sequence at $n=1$ and sum all the terms up to $n=3$

$$\sum_{n=1}^{3} u_n = u_1 + u_2 + u_3 = 153$$

$$3 + b - 3a + b - ab + 3a^2 = 153 \qquad \boxed{b = a+9}$$

$$3 + a + 9 - 3a + a + 9 - a(a+9) + 3a^2 = 153$$

$$21 - a - a^2 - 9a + 3a^2 = 153$$

$$2a^2 - 10a + 21 = \cancel{153}$$

$$-153 \qquad -153$$

$$[2a^2 - 10a - 132 = 0] \div 2$$

$$a^2 - 5a - 66 = 0$$

(c) Factorise $a^2 - 5a - 66 = 0$:

$$(a+6)(a-11) = 0$$

POSSIBLE VALUES OF a :

$a = -6$

$a = 11$

$$u_2 = b - 3a = a + 9 - 3a$$
$$= -2a + 9$$

$$u_2 = -2(-6) + 9 = 21$$

$$u_2 = -2(11) + 9 = -13$$

Question 11 continued

$21 > -13$

So $u_2 = 21 \rightarrow$ larger value!

(Total for Question 11 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS